MATHSPEAK:

A Talk on Verbalizing Math by Dr. Abraham Nemeth, Creator of the Nemeth Math Braille Code

When I was studying math at the college and post graduate levels, I used sighted readers for accessing text materials, since braille materials in math were almost entirely unavailable. Even as a college professor, particularly during my early years, I continued to use sighted readers, volunteers, paid, and teaching assistants, largely for the same reason.

No standard protocol exists for articulating mathematical expressions as it does for articulating the words of an English sentence. Therefore, it was necessary for my reader and me to come to some agreement as to the most efficient method for conveying mathematical text to me in an unambiguous manner. Since I needed to turn in assignments as a student, and since I often prepared handouts for my students and for faculty seminars as a professor, it soon became clear that the same protocol should be used when I was dictating and my reader was doing the writing. Little by little, a surprisingly simple protocol evolved. I can teach it to a new reader in about 15 minutes. The most important feature of this protocol is the principle that anything that was read earlier or anything that will be read later should never affect what I write now in response to my reader's current utterance. I appropriated this principle from the Nemeth Code which I had already developed and which was, of course, the code that I was using for the writing of mathematical text.

At our recent convention in Chicago, this matter of "mathspeak" came up at our Science ane Engineering meeting. At the conclusion of this item, John Miller, our Chairman, gently but firmly instructed me to prepare a writeup of this method and post it on the Internet. What follows is my attempt to comply. I am posting it on the R&D "listserve" and am asking John to forward it to the S&E listserve since I do not know the E-mail adress of the latter. And, John, if you will give me that address, I will subscribe.

It had never occurred to me before now that the method was sufficiently substantial to warrant a formal description in writing. Without further ado, here's how "mathspeak" works.

Letters:

Lowercase letters are pronounced at face value without modification. They are never combined to form words. In particular, the trigonometric and other function abbreviations are spelled out rather than pronounced as words. We say "s i n" rather than "sine," "t a n" rather than "tan" or "tangent," "l o g" rather than "log," etc.

A single uppercase letter is spoken as "upper" followed by the name of the letter. If a word is in uppercase, it is spoken as "upword" followed by the sequence of letters in the word, pronounced one letter at a time.

For a Greek letter, my reader is taught to say "Greek" followed by the English name of the letter. If he knows its Greek name, he simply speaks that name. Thus, my reader might say "Greek e" or "epsilon." Uppercase Greek letters are pronounced as "Greek upper" followed by the English name of the letter, or "upper" followed by the name of the Greek letter. For new readers, I had a card with the lowercase and the uppercase Greek letters and their names. Pretty soon my readers did not have to refer to the card, or they did so only for infrequently occurring Greek letters.

I never devised a formal method for pronouncing letters that were printed in italic type, boldface type, script type, san serif type, or that were underlined. However, I do not anticipate any difficulties in this area, should the need for a formal "mathspeak" protocol become desirable.

Digits and Punctuation:

Digits are pronounced individually; never as words. Thus, 15 is pronounced "1 5" and not "fifteen." Similarly, 100 is pronounced "1 0 0" and not "one hundred." An embedded comma is pronounced "comma," and a decimal point, whether leading, trailing, or embedded, is pronounced "point."

The period, comma, and colon are pronounced at face value as "period," "comma," and "colon." Other punctuation marks have longer names and are pronounced in abbreviated form. Thus, the semicolon is pronounced as "semi," and the exclamation point is pronounced as "shriek." I borrowed the latter from the APL programming language in which "shriek" is the standard way of referring to the exclamation mark.

The grouping symbols are particularly verbose and require abbreviated forms of speech. Thus, we say "L-pare" for the left parenthesis, "R-pare" for the right parenthesis, "L-brack" for the left bracket, "R-brack" for the right bracket, "L-brace" for the left brace, "R-brace" for the right brace, "L-angle" for the left angle bracket, and "R-angle" for the right angle bracket.

Operators and Other Math Symbols:

We say "plus" for plus and "minus" for minus. We say "dot" for the multiplication dot and "cross" for the multiplication cross. We say "star" for the asterisk and "slash" for the slash.

We say "superset" in a set-theoretic context or "implies" in a logical context for a left-opening horseshoe. We say "subset" for a right-opening horseshoe. We say "cup" (meaning union) for an up-opening horseshoe and "cap" (meaning intersection) for a down-opening horseshoe.

We say "less" for a right-opening wedge and "greater" for a left-opening wedge. We say "join" for an up-opening wedge and "meet" for a down-opening wedge. The words "cup," "cap," "join," and "meet" are standard mathematical vocabulary.

We say "less-equal" and "not-less" when the right-opening wedge is modified to have these meanings. We say "greater-equal" and "not-greater" under similar conditions for the left-opening wedge. We say "equals" for the equals sign and "not-equal" for a cancelled-out equals sign. We say "element" for the set notation graphic with this meaning, and we say "contains" for the reverse of this graphic. We say "partial" for the round d, ane we say "del" for the inverted uppercase delta.

We say "dollar" for a slashed s, "cent" for a slashed c, and "pound" for a slashed l.

We say "integral" for the integral sign. We say "infinity" for the infinity sign, and "empty-set" for the slashed 0 with that meaning. We say "degree" for a small elevated circle, and we say "percent" for the percent sign. We say "ampersand" for the ampersand sign, and "underbar" for the underbar sign. We say "crosshatch" for the sign that is sometimes called number sign or pound sign. We say "space" for a clear space in print.

Although the above is a long list, it is not intended to be complete. It does, however, cover most of the day-to-day material with which we usually deal.

Fractions and Radicals

We say "B-frac" as an abbreviation for "begin-fraction," and "E-frac" as an abbreviation for "end-fraction". We say "over" for the fraction line. Even the simplest fractions require "B-frac and E-frac. Thus, to pronounce the fraction "one-half" according to this protocol, we say "B-frac 1 over 2 E-frac." By this convention, a fraction is completely unambiguous. If we say "B-frac a plus b over c + d E-frac," the extent of the numerator and of the denominator are completely unambiguous.

A simple fraction (which has no subsidiary fractions) is said to be of order 0. By induction, a fraction of order n has at least one subsidiary fraction of order n-1. A fraction of order 1 is frequently referred to as a complex fraction, and one of order 2 as a hypercomplex fraction. Complex fractions are fairly common, hypercomplex fractions are rare, and fractions of higher order are practically non-existent. The order of a fraction is readily determined by a simple visual inspection, so that the sighted reader forms an immediate mental orientation to the nature of the notation with which he is dealing. It is important for a Braille reader to have this same information at the same time that it is available to the sighted reader. Without this information, the Braille reader may discover that he is dealing with a fraction whose order is higher than he expected, and may have to reformulate his thinking accordingly long after he has become aware of the outer fraction.

To communicate the presence of a complex fraction, we say "B-B-frac," "O-over," and "E-E-frac" for the components of a complex fraction, in the manner of stuttering. For a hypercomplex fraction, the components are spoken as "B-B-B-frac," "O-O-over," and "E-

E-E-frac," respectively. You can see that the speech patterns are designed to facilitate transcription in the Nemeth Code, according to the rules of that Code.

Radicals are treated much like fractions. We say "B-rad" and "E-rad" for the beginning and the end of a radical, respectively. Thus, we say "B-rad 2 E-rad for the square root of 2.

Nested radicals are treated just like nested fractions, except that there is no corresponding component for "over." Thus, if we say "B-B-rad a plus B-rad a plus b E-rad plus b E-E-rad," the braille reader is immediately alerted to the structure of the notation just as the sighted reader is by mere inspection, and the expression is unambiguous.

Subscripts and Superscripts

We introduce a subscript by saying "sub," and a superscript by saying "sup" (pronounced like "soup.") We do not say "x square;" instead we say "x sup 2." We say "base" to return to the base level. The formula for the Pythagorean Theorem would be spoken as "z sup 2 base equals x sup 2 base plus y sup 2 base period."

Whenever there is a change in level, the path, beginning at the base level and ending at the new level, is spoken. Thus, if e has a superscript of x, and x has a subscript of i+j, we say "e sup x sup-sub i plus j." And if e has a superscript of x, and x has a superscript of 2, we say "e sup x sup-sup 2." If the superscript on e is x square plus y square, we say "e sup x sup-sup 2 sup plus y sup-sup 2." If an element carries both a subscript and a superscript, we speak all of the subscript first and then all of the superscript. Thus, if e has a superscript of x, and x has a subscript of i+j and a superscript of p sub k, we say "e sup x sup-sub i plus j sup-sup p sup-sup-sub k."

If a radical is other than the square root, we speak the radical index as a superscript to the radical. Thus, the cube root of x+y is spoken as "b-rad sup 3 base x plus y E-rad."

We say "underscript" for a first-level underscript, and we say "overscript" for a first level overscript. We say "endscript" when all underscripts and overscripts terminate. Thus we say "upper sigma underscript i equals 1 overscript n endscript a sub i." We say "un-underscript" and "O-overscript" for a second-level underscript and a second-level overscript, respectively. We speak all the underscripts in the order of descending level before speaking any of the overscripts. Each level is preceeded by "underscript" with the proper number of "un" prefixes attached. Similarly, we speak the overscripts in the order of ascending level. Each level is preceeded by "overscript" with the proper number of "O" prefixes attached.

Conclusion:

The speech generated by this protocol is not exactly what a professor in class would use, but it is absolutely unambiguous and results in a perfect Nemeth Code transcription. It avoids largely unsuccessful attempts by a reader to describe the notation he sees, accompanied by the shouting and gesturing that such attempts at description engender. When each of a number of readers abides by this protocol, it is a snap to record the information, and makes much better use of the time spent with a reader.

As Raised Dot Computing has amply demonstrated, Nemeth Code can be converted into correctly formatted print notation. And this print notation can be converted into speech by using the protocol described in this paper. Thus, speech to Nemeth Code to print to speech demonstrates that these three systems are notationally equivalent. The ability to convert from one form to another is a distinct benefit to a blind person engaged in the field of mathematics, whether as a student, a teacher, or as a worker in a professional field, and gives him a competitive edge.

These considerations strongly suggest that serious thought be given to refining "mathspeak" and making it a standard by which complex mathematical notation can be communicated to a blind person in verbal form. It could also serve as the basis for transmitting mathematical notation electronically when ASCII is not capable of conveying the notation (many math symbols have no ASCII representation) or when ASCII codes which represent notation are likely to be unfamiliar to the recipient.