# MAT 271 Section 3.7 Rates of Change in the Natural and Social Sciences Guided Notes Filled in Answer Key

## Learning Outcomes:

The learner will be able to examine some applications with regards to physics, chemistry, biology, economics, and other sciences.

## Rates of Change

Graph displaying curved segment of the graph of a function f(x)  drawn in the first quadrant with a line segment tangent to it at the point (x1, f(x1)). 
 Also labeled on the graph of f(x) to the right of (x1,f(x1)) is the point (x2,f(x2)).  The distance along the x axis between x1 and x2 is the quantity delta-x.  The distance along the y-axis between y1=f(x1) and y2=f(x2) is the quantity delta-y.
The secant line segment between (x1,f(x1)) and (x2,f(x2)) is the hypotenuse of a right triangle with sides of length delta-x and delta-y.  The slope of the line segment tangent to the graph of f(x) at x1 is  the Instantaneous rate of change of f at x1 and is approximated by the slope of the secant line segment which is the average rate of change of f from x1 to x2.

Graph description: Graph displaying curved segment of the graph of a function f(x) drawn in the first quadrant with a line segment tangent to it at the point .

Also labeled on the graph of  to the right of  is the point . The distance along the x axis between  and  is the quantity delta-x. The distance along the y-axis between  and  is the quantity delta-y.

The secant line segment between  and is the hypotenuse of a right triangle with sides of length delta-x and delta-y. The slope of the line segment tangent to the graph of  at  is the Instantaneous rate of change of f at  and is approximated by the slope of the secant line segment which is the average rate of change of f from  to .

Recall from section 2.7 the change in  is, and the corresponding change in  is . Additionally, the difference quotient, , is the **average** rate of change of with respect to . Furthermore, is the **instantaneous** rate of change of with respect to .

## Motion

Suppose an object moves along a straight line according to an equation of motion , where  is the **displacement** (directed distance) of the object from the origin at time . Then is the instantaneous **velocity**, and is the **acceleration**. This was discussed in chapter 2, but with the differentiation formulas we can save time and answer more questions.

### Example

The position of a particle is given by the equation  where  is measured in seconds and in meters.

1. Determine the velocity at time .



1. Determine the velocity after 2 seconds and after 4 seconds.

Evaluate at  and  seconds respectively.

meters per second

 meters per second

1. When is the particle at rest?

The particle is at rest when the velocity is zero.













1. When is the particle moving forward (positive direction)?

The particle is moving in a positive direction when the velocity is greater than zero.



Divide a number line and put the zeros of the velocity on it. Namely the zeros are 1 second and three seconds. This divides the number line into these three regions: . Pick an x value to test in each region and evaluate where v(t) is positive or negative. The final answer is: 

1. Sketch a diagram representing the motion of the particle.

The particle moves in a positive direction (left to right) from  seconds to  second. At  second, the particle changes direction and moves in a negative direction (right to left) until  seconds. At seconds, the particles switches directions again moving back in the positive direction (left to right) after seconds.

1. Determine the distance traveled by the particle during the first five seconds.

You must add the distances during the time values below:



meters

meters

meters

Add these values 4 + 4 + 20 for a total distance traveled of 28 meters.

1. Determine the acceleration at time  and after 4 seconds.

Remember that acceleration isn’t the same as speed. If velocity and acceleration have the same sign at a t value, then the object is speeding up at that time. If velocity and acceleration have opposite signs at a t value, then the object is slowing down at that time.







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Graph description: Graph of two curves and one line drawn on the horizontal interval 0 to 5 and vertical interval -15 to 20. The horizontal axis represents time elapsed since a particle is set in motion, in seconds

The curve labeled "s" represents the position of the particle at time t. It is a segment of a polynomial and passes through the points (0,0), (1,5), (3,0), (4,3) and (5,20).

The curve labeled "v" represents the velocity of the particle at time t. It is a segment of a parabola that opens up with a vertex at (2,-3), It passes through the points (0,9), (1,0), (2, -3), (3,0), (4,10) and (5,25).

The line segment is labeled "a" and represents the acceleration of the particle. It passes through the points (0,-12),(1,-6),(2,0),(3,6), (4,12) and (5,18).

The time intervals from 0 to 1 second, and 3 to 5 seconds are labeled "forward" and the interval from 1 to 3 seconds is labeled "backward".

The time intervals from 0 to 1 second and 2 to 3 seconds are labeled "slows down" and the time intervals from 1 to 2 seconds and from 3 to 5 seconds are labeled "speeds up".

### Example:

A particle moves with position function .

1. At what time does the particle have a velocity of ?

Take the velocity function and set it equal to 20 and solve for t.













 seconds is extraneous

1. At what time is the acceleration 0? What is the significance of this value?

Set the acceleration function equal to zero and solve for t.





Using the quadratic formula, you obtain . The value  is extraneous.

When the acceleration is zero, the velocity is constant.

### Example:

A spherical balloon is being inflated. Determine the rate of increase of the surface area () with respect to . Then determine the rate of increase of the surface area when foot, feet, and feet.

The rate of increase in the surface area requires taking the derivative with respect to the variable r.











## Marginal Cost

Suppose is the total cost that a company incurs in producing units of a certain commodity. The function  is called the **cost** function. If the number of items produced is increased from to , then the additional cost is , and the average rate of change of the cost is . The limit of this quantity as  (the **instantaneous** rate of change with respect to the number of items **produced**) is called the **marginal** cost.

So marginal cost .

### Example:

Suppose that the cost (in dollars) for a company to produce pairs of tennis shoes is .

1. Determine the marginal cost function.



1. Determine  and explain its meaning.



It costs $65 to produce the 101st item.

1. Compare  with the cost of manufacturing the 101st pair.



The derivative  is approximately the same as the cost of producing the 101st pair.

## Chemistry

* Instantaneous rate of reaction is the limit of the average rate of reaction as the time interval approaches 0. 
* Isothermal compressibility  which measures how fast the volume of a substance decreases as the pressure on it increases at a constant temperature.

## Biology

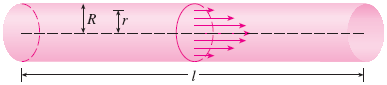
* Growth rate , where  is the number of individuals of a population at time 
* Although the above growth rate is not exactly true as we would actually have a step function for population growth, we obtain approximations to determine a curve to fit the data as shown below. Graph of a step function with  a logistic curve drawn through it.

  The step function consists of 24 horizontal line segments that are arranged along the path of a logistic curve.  The left endpoint of each line segment has a closed endpoint on the left and an open endpoint on the right.  The left endpoint of each succeeding line segment lies directly above the preceding segment's right endpoint.   The line segments are of decreasing width as they move from left to right along the logistic curve.  

Graph description: Graph of a step function with a logistic curve drawn through it.

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* Law of laminar flow where is the viscosity of the blood and is the pressure difference between the ends of the tube.



Graph description: A tube oriented horizontally with outer radius R and inner radius r, of length l.

A line is drawn laterally through the center of the tube.

A group of 7 parallel arrows of diminishing length are drawn laterally along the tube indicating the direction of flow through the tube. The longest arrow runs directly through the center of the tube and the length of the arrows decreases from the center radially on either side of the longest arrow so that shortest arrows are directly next to the sides of the tube.

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Stewart, J. (2016). *Calculus, Early Transcendentals*. (8th ed.). Cengage Learning: Boston, MA.